$$\lambda_n = K / \alpha'_n, \quad \alpha_n' = K / \lambda_n$$

and for a negative value of λ_n we have

$$\lambda_n = -K / \alpha_n''$$

from which it follows that $\alpha_n'' = -K / \lambda_n$.

The initial function $p_0(x)$ which represents the pressure at the initial instant of time, can be expanded into a series in eigenfunctions of the integral equation (2.6) which are orthogonal by virtue of the kernel

$$p_0(x) = \sum_{n=1}^{\infty} A_n p_{n'}(x) + \sum_{m=1}^{\infty} B_m p_{m''}(x)$$

The pressure at subsequent instants of time is given by

$$p(x,t) = \sum_{n=1}^{\infty} A_n \exp\left(-\frac{K}{|\lambda_n'|} t\right) p_{n'}(t) + \sum_{m=1}^{\infty} B_m \exp\left(-\frac{K}{|\lambda_m''|} t\right) p_{m''}(t)$$

where λ_n' denotes the positive, and λ_m'' the negative characteristic numbers.

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Translated by L. K.

UDC 62-50

AN ALTERNATIVE IN THE DIFFERENTIAL-DIFFERENCE GAME OF APPROACH – EVASION WITH A FUNCTIONAL TARGET

PMM Vol. 40, № 6, 1976, pp. 987-994

V. I. MAKSIMOV
(Sverdlovsk)

(Received June 7, 1976)

An approach-evasion problem with a functional target set under constraints on the system's trajectory is studied for a conflict-controlled system described by a differential-difference equation. The main result states: either a strategy exists for the first player resolving the approach problem or a strategy exists for the second player resolving the evasion problem. The paper is closely related to [1–6].

1. We examine the system with aftereffect

$$x^{\cdot}(t) = f(t, x_t(s), u, v), \quad t_0 \leqslant t \leqslant \vartheta \tag{1.1}$$

$$u \in P \subset E_{r_1}, \quad v \in Q \subset E_{r_2}$$

Here x is the n-dimensional phase vector; u and v are the controls of the first and second players; P and Q are compacta; the functional $f(t, x_t(s), u, v)$ is defined on the product $[t_0, \vartheta] \times B_\omega \times P \times Q$, where B_ω is any one of the spaces $C([-\omega, 0])$, or H_ω ($w =$ const $\geqslant 0$); $C([-\omega, 0])$ is the space of continuous n-dimensional functions $x(s)$, $-\omega \leqslant s \leqslant 0$, with the norm $\|x(s)\|_{1\omega} = \max_s \|x(s)\|$; H_ω is the Hilbert space of the n-dimensional functions $x(s)$ with the norm

$$\|x(s)\|_{2\omega} = (\|x(0)\|^2 + \int_{-\omega}^{0} \|x(s)\|^2 ds)^{1/2}$$

$$\|z\| = (z_1^2 + z_2^2 + \ldots + z_n^2)^{1/2}, \qquad z \in E_n$$

$$f(t, x(s), u, v) = f^*(t, x(-\tau_1), \ldots, x(-\tau_m), \varphi(t, x(s)), u, v)$$

when $B_\omega = H_\omega$. The functionals $f(t, x(s), u, v)$, $\varphi(t, x(s))$ and $f^*(t, z_1, \ldots, z_m, z, u, v)$ are continuous and satisfy a Lipschitz condition in $x(s)$ and $(z_1, \ldots, z_m, z)$, respectively. The following growth conditions are satisfied uniformly with respect to $u \in P$ and $v \in Q$ for any $x(s) \in B_\omega$: if $B_\omega = C([-\omega, 0])$, then

$$\|f(t, x(s); u, v)\| \leqslant \zeta_1(t) + \zeta_2(t)\|x(s)\|_{1\omega}$$

if $B_\omega = H_\omega$, then

$$\|f^*(t, z_1, \ldots, z_m, \varphi(t, x(s)), u, v)\| \leqslant \zeta_1(t) + \zeta_2(t)\|x(s)\|_{2\omega} + \sum_{j=1}^{m} \eta_j(t)\|z_j\|$$

where $\zeta_i(t)$ are nonnegative functions summable on $[t_0, \vartheta]$; $\eta_j(t)$ are nonnegative functions square-summable on $[t_0, \vartheta]$. We denote

$$x(s; \delta) = x(s), \ x_t(s; \delta) = x(t + s; \delta), -\delta \leqslant s \leqslant 0 \ (\delta \geqslant 0)$$

The element $x_0(s; \omega)$ and the nonempty closed sets

$$N \subset [t_0, \vartheta] \times B_\omega, \ M \subset [t_0 - \omega + \tau, \vartheta] \times B_\mu \ (\mu = \text{const} \geqslant 0, \tau = \max[\omega, \mu])$$

are specified.

The purpose of choosing the control u, formed on the feedback principle $u[t] = u(t, x(t + s; \tau))$, is to ensure that the trajectory segment $x(t + s; \mu)$ is led onto M no later than at the instant ϑ while retaining the trajectory segments realized during the motion in a given set N for any measurable realization $v[t]$. The purpose of choosing the control v, formed on the feedback principle $v[t] = v(t, x(t + s; \tau))$, is to ensure that the motion evades set M inside N or that this motion is led out of N before the instant it hits onto M. Similar problems for systems with aftereffect were studied in [2, 3] wherein the corresponding alternative statements were proved. In the present paper, in contrast to [2, 3], these problems are satisfied under the additional assumptions that functional phase constraints are present and that the target sets are more general.

The rule which associates a nonempty closed set $U(p) \subset P$ with each pair $p = \{t, x(s; \tau)\}$, $t \geqslant t_0$, i.e., the game's positions, is called the first player's strategy U For $\mu > \omega$ and $t = t_0$ we set

$$x(s; \tau) = x_0(s; \omega), \ s \in [-\omega, 0]$$

$$x(s; \tau) = x_0(-\omega), \ s \in [-\tau, -\omega))$$

Let Δ be some finite covering of the interval $[t_0, \vartheta]$ by the intervals $\tau_i \leqslant t < \tau_{i+1}$, $\tau_0 = t_0$, $i = 0, 1, \ldots$; $\delta = \max_i (\tau_{i+1} - \tau_i)$. By $x_\Delta[t; p_0, U]$ we denote the function $x_\Delta[t]$, absolutely continuous at $t_0 \leqslant t \leqslant \vartheta$, such that $x_\Delta[t_0 + s; \tau] = x_0(s; \tau)$ and for almost all $t \in [t_0, \vartheta]$

$$x_\Delta^\cdot[t] \in F(t, x_\Delta[t + s; \omega], u[\tau_i])$$

where

$$u[\tau_i] \in U(\tau_i, x_\Delta[\tau_i + s; \tau]), F(t, x_\Delta[t + s; \omega]$$

$$u[\tau_i]) = \overline{\text{co}}\, \{f(t, x_\Delta[t + s; \omega], u[\tau_i], v)\ v \in Q\}$$

A continuous function with the following property is called a motion $x[t] = x[t, p_0, U]$: a sequence $\{\Delta_j\}$ of coverings with $\{\delta_j\} \to 0$ exists such that $\{x_{\Delta_j}[t; p_0, U]\}$ converges in $C([t_0, \vartheta])$ to $x[t]$ for some sequence of functions. The strategy V and the motions $x[t; p_0, V]$ and $x[t; p_0, U, V]$ are defined analogously. We denote: $X(\cdot; p_0, V)$ is the sheaf of all motions $x[t] = x[t; p_0, V]$ for $t \in [t_0, \vartheta]$ and $X_\delta(t_* + s; p_0, V) = \{x_{t_*}(s; \delta) \mid x(t) = x[t; p_0, V]\}$. The following statement is valid [2, 3]:

Lemma 1.1. The sets $X(\cdot; p_0, U)$, $X(\cdot; p_0, V)$ and $X(\cdot; p_0, U, V)$ are compact in themselves in $C([t_0, \vartheta])$ and the inclusions

$$X(\cdot; p_0, U, V) \subset X(\cdot; p_0, U)$$
$$X(\cdot; p_0, U, V) \subset X(\cdot; p_0, V)$$

hold.

Problem 1.1. Find the strategy $U^e(t, x(s; \tau))$ which ensures the contact

$$\{\tau_*, x[\tau_* + s; \mu]\} \in M$$
$$\{t, x[t + s; \mu]\} \notin M, \quad t_0 + \tau - \omega \leqslant t < \tau_*$$
$$\{\xi, x[\xi + s; \omega]\} \in N, \quad t_0 \leqslant \xi < \tau_* \leqslant \vartheta$$

for every motion $x[t] = x[t; p_0, U^e]$ for $\tau_* \geqslant t_0 + \tau - \omega$.

Problem 1.2. Find the neighborhood $H(N)$ and $G(M)$ and the strategy $V^e(t, x(s; \tau))$ which exclude the contact

$$\{\tau_*, x[\tau_* + s; \mu]\} \in G(M)$$
$$\{t, x[t + s; \omega]\} \in H(N), \quad t_0 \leqslant t < \tau_* \leqslant \vartheta$$

for every motion $x[t] = x[t; p_0, V^e]$ for $\tau_* \geqslant t_0 + \tau - \omega$.

Problem 1.3 (1.4). Solve Problem 1.1 (1.2) with $\tau_* = \vartheta$.

2. We give below a method for solving Problems 1.1 — 1.4, based on the concept of extremal strategy from [2 — 4]. In what follows we assume the satisfaction of

Condition A. The equality

$$\min_{u \in P}\max_{v \in Q} l' f(t, x(s), u, v) = \max_{v \in Q}\min_{u \in P} l' f(t, x(s), u, v)$$

is valid for any vector $l \in E_n$ and pair $\{t, x(s)\} \in [t_0, \vartheta] \times B_\omega$.

Let some set $W(t) \subset B_\tau$ be associated with each value of parameter t $(t_0 \leqslant t \leqslant \vartheta)$. Following [1, 2] we say that a set $W(t)$, $t_0 \leqslant t \leqslant \vartheta$ is u-stable (v-stable) if for any numbers $t_0 \leqslant t_* < t^* \leqslant \vartheta$, element $x_*(s; \tau) \in W(t_*)$ and strategy V (U) the motion

$$x_V[t] = x[t; \{t_*, x_*(s; \tau)\}, V]$$
$$(x_U[t] = x[t; \{t_*, x_*(s; \tau)\}, U])$$

exists such that

$$x_V\,[t^* + s;\ \tau] \in W\,(t^*)\,(x_U\,[t^* + s;\ \tau] \in W\,(t^*))$$

or

$$\{t,\ x_V\,[t + s;\ \mu]\} \in M\ (\{t,\ x_U\,[t + s;\ \omega]\} \notin H\,(N))$$

even if for one $t \in [t_*,\ t^*]$. In what follows we also use the concept of strongly stable sets from [2].

We choose an arbitrary element $x\,(s) \in B_\tau$ and for $W\,(t) \neq \varnothing$ we set

$$r\,(x\,(s),\ W\,(t)) = \inf_{y(s)\in W(t)} \| x\,(s) - y\,(s) \|_\tau \tag{2.1}$$

($\|\cdot\|_\tau$ is the norm in B_τ).

A strategy U^e is said to be extremal to sets $W\,(t)$, $t_0 \leqslant t \leqslant \vartheta$, if it is given by sets $U_e\,(t,\ x\,(s;\ \tau))$ of the form: if $W\,(t) = \varnothing$, then $U_e\,(t,\ x\,(s;\ \tau)) = P$; if $W\,(t) \neq \varnothing$, then

$$U_e\,(t,\ x\,(s;\ \tau)) = \{u_e \,|\, \max_{v\in Q}\,(x\,(0) - z)'\,f\,(t,\ x\,(s;\ \omega),\ u_e,\ v) = \min_{u\in P}\,\max_{v\in Q}\,(x\,(0) - z)'\,f\,(t,\ x\,(s;\ \omega),\ u,\ v)\}$$

Here z is an arbitrary element of set $Z\,(t,\ x\,(s;\ \tau))$; $Z\,(t,\ x\,(s;\ \tau))$ is the collection of elements closest to $x\,(0)$ in E_n from the set of partial limits of the sequence $\{x^{(k)}\,(0)\}$ which is the 0-section [2] of some section $\{y\} = \{x^{(k)}\,(s;\ \tau)\}$ from (2.1), being a minimizing sequence for $x\,(s;\ \tau)$; the prime denotes transposition. The strategy V^e is defined analogously. We have the valid

L e m m a 2. 1. Let the initial position $p_0 = \{t_0,\ x_0\,(s;\ \tau)\}$ be such that

$$x_0\,(s;\ \tau) \in W\,(t_0) \tag{2.2}$$

If the sets $W\,(t)$, $t_0 \leqslant t \leqslant \vartheta$ are strongly $u\,(v)$-stable, then the strategy $U^e\,(V^e)$ extremal to them ensures the equality

$$r\,(x_t\,[s;\ \tau],\ W\,(t)) = 0,\ t_0 \leqslant t \leqslant \vartheta$$

for any $x\,[t] = x\,[t;\ p_0,\ U^e]\ (x\,[t] = x\,[t;\ p_0,\ V^e])$.

We denote: $W_\delta\,(t) = \{x\,(s) \in H_\delta \,|\, x\,(0) = y\,(0),\ x\,(s) = y\,(s;\ \tau)$ for almost all $-\delta \leqslant s \leqslant 0,\ y\,(s;\ \tau) \in W\,(t)\}$; $\overline{K}$ is the closure of set K; $X_\delta\,(t_2,\ x\,(s;\ \tau),\ t_1,\ V) = \{y = \{t,\ z\,(s)\} \,|\, t \in [t_1,\ t_2],\ z\,(s) = x\,[t + s;\ \delta],\ x\,[\cdot] \in X\,(\cdot;\ \{t_1,\ x\,(s;\ \tau)\},\ V)\}$; $M\,(t_*)$ $(N\,(t_*))$ is the section of set M (N) by the hyperplane $t = t_*$; $W = \{\{t,\ x\,(s;\ \tau)\} \,|\, t \in [t_0,\ \vartheta],\ x\,(s;\ \tau) \in W\,(t)\}$.

In a way similar to Theorem 2. 2 of [4] we can prove

L e m m a 2. 2. Let u-stable nonempty sets $W\,(t)$, $t_0 \leqslant t \leqslant \vartheta$ exist such that $W_\omega\,(t) \subset N\,(t)$ for all $t \in [t_0,\ \vartheta]$ and $W_\mu(\vartheta) \subset M\,(\vartheta)$. Then, when conditions (2. 2) are satisfied the strategy U^e extremal to these sets solves Problem 1. 1.

There holds

L e m m a 2. 3. Let the nonempty sets $W\,(t)$, $t_0 \leqslant t \leqslant \vartheta$ be v-stable and let a closed neighborhood $G_1\,(M)$ of set M exist with the property: $W \cap G_1\,(M) = \varnothing$. Then we can find a number $\alpha > 0$ such that the strategy V^e extremal to them solves Problem 1. 2 for all $x_0\,(s;\ \tau)$ from the α-neighborhood of set $W\,(t_0)$.

On the basis of Lemmas 1. 1 and 2. 1 we can verify the validity of

L e m m a 2. 4. Let the system of sets $D\,(t) \subset B_\tau$, $t_0 \leqslant t \leqslant \vartheta$ be $u\,(v)$-stable; then the system of sets $\overline{D}\,(t)$, $t_0 \leqslant t \leqslant \vartheta$ is $u\,(v)$-stable.

From the space $\{t, x(s;\tau)\}$ we remove all those positions $\{t_*, x_*(s;\tau)\}$ $(t_* \in [t_0, \vartheta])$ for each of which Problem 1.2 (1.4) is solvable on the interval $[t_*, \vartheta]$. By $W^u(t_*)$ $(W^\circ(t_*))$ we denote the set, closed in B_τ, of all the remaining positions.

Lemma 2.5. The sets $W^u(t)$, $t_0 \leqslant t \leqslant \vartheta$ are u-stable. If $x_0(s;\tau) \in W^u(t_0)$, then the strategy U^e extremal to them solves Problem 1.1.

This assertion can be proved along the lines of the proofs of the analogous assertions in [1 — 3]. Assuming that the sets $W^u(t)$, $t_0 \leqslant t \leqslant \vartheta$ are not u-stable, we conclude: we can find $t_* \in [t_0, \vartheta]$, $t^* \in (t_*, \vartheta]$, $x_*(s;\tau) \in W^u(t_*)$ and a strategy V_1 such that

$$X_\tau(t^* + s; \{t_*, x_*(s;\tau)\}, V_1) \cap W^u(t^*) = \varnothing$$

$$X_\mu(t^*, x_*(s;\tau), t_*, V_1) \cap M = \varnothing$$

By the definition of sets $W^u(t)$, with certain $V = V(x_k)$, $G(M) = G_k(M)$ and $H(N) = H_k(N)$ Problem 1.2 is solvable for every $x_k(s;\tau) \in X_\tau(t^* + s; \{t_*, x_*(s;\tau)\}, V_1)$. We consider the set

$$W(x_k) = \{\{t, x(s;\tau)\} \mid t^* \leqslant t \leqslant \tau_k(x[\cdot])\}$$

$$x(s;\tau) = x_\tau[t + s; \{t^*, x_k(s;\tau)\}, V(x_k)]$$

where $(\tau_k(x[\cdot])$ is the instant at which the motion $x[t]$ first leaves the region $H_k(N)$ It can be proved that the sets $W_0(t, x_k)$, $t^* \leqslant t \leqslant \vartheta$, where $W_0(t, x_k) = W(t, x_k)$ if $W(t, x_k) \neq \varnothing$ and $W_0(t, x_k) = \varnothing$ if $W(t, x_k) = \varnothing$, are v-stable. Using Lemma 2.3 we easily verify that the strategy $V^e(x_k)$ extremal to $W_0(t, x_k)$, $t^* \leqslant t \leqslant \vartheta$, solves Problem 1.2 for certain $\beta_k > 0$, $G_k^*(M)$ and $H_k^*(N)$ for all $x(s;\tau) \in S(\beta_k)$. ($S(\beta_k)$ is a neighborhood of radius β_k in B_τ of the element $x_k(s;\tau)$). By virtue of the compactness in B_τ of the set $X_\tau(t^* + s; \{t_*, x_*(s;\tau)\}, V_1)$ (Lemma 1.1), it can be covered by a finite system of such neighborhoods $S(\beta_k)$ $k = 1, 2, \ldots, q$. We consider the set $W_1 = \{W_1(t), t_* \leqslant t \leqslant \vartheta\}$ of pairs $\{t, x(s;\tau)\}$, $x(s;\tau) \in W_1(t)$, which satisfy the condition

$$t_* \leqslant t \leqslant t^*, \; x(s;\tau) = x_\tau[t + s; \{t_*, x_*(s;\tau)\}, V_1]$$

or the condition

$$t^* \leqslant t \leqslant \tau^*(x[\cdot]), \; x(s;\tau) = x_\tau[t + s; \{t^*, x(s;\tau)\}, V^e(x_k)]$$

Here $x(s;\tau)$ is an arbitrary element of $S(\beta_k)$; $\tau^*(x[\cdot])$ is the instant at which the motion $x[t]$ first leaves the region

$$H(N) = \bigcap_k H_k^*(N).$$

Let us prove the v-stability of sets $W_1(t)$, $t_* \leqslant t \leqslant \vartheta$. We assume to the contrary that instants t_1 and t_2 $(t_2 > t_1)$ from $[t_*, \vartheta]$, an element $x_1(s;\tau) \in W_1(t_1)$ and a strategy U_1 exist such that

$$\{t, x_\omega[t + s; \{t_1, x_1\}, U_1] \in H(N), \; t_1 \leqslant t \leqslant t_2 \tag{2.3}$$

$$X_\tau(t_2 + s; \{t_1, x_1(s;\tau)\}, U_1) \cap W_1(t_2) = \varnothing \tag{2.4}$$

For instance, let $t_1 > t^*$. From the method of constructing W_1 we can find a number k_1, an element $g(s;\tau) \in S(x_{k_1})$ and a motion $x^\circ[t] = x[t; \{t^*, g(s;\tau)\}, V^e(x_{k_1})]$ such that $x_1(s;\tau) = x^\circ[t + s;\tau]$, $t_1 \leqslant \tau^*(x^\circ[\cdot])$. We consider an arbitrary motion

$$x^*[t] = x[t; \{t_1, x_1(s;\tau)\}, U_1, V^e(x_{k_1})]$$

If $\tau^* = \tau^*(x^*[\cdot]) \leqslant t_2$, then by the construction of $W^u(t)$ and Lemma 1.1, $\{\tau^*, x^* \times [\tau^* + s; \omega]\} \notin H(N)$, which contradicts (2.3). If $\tau^* > t_2$, then $x^*[t_2 + s; \tau] \in W_1(t_2)$, which contradicts (2.4). The v-stability of the sets $W_1(t)$, $t_* \leqslant t \leqslant \vartheta$ and with $t_1 \leqslant t^*$, can be verified analogously. It is clear that $W_1 \cap \overline{G}(M) = \varnothing$, for any closed neighborhood of the set $M\ \overline{G}(M) \subset \bigcap_k \overline{G}_k^*(M)$. Using Lemma 2.3 we can establish (since $x_*(s;\tau) \in W_1(t_*)$) that the strategy V^e extremal to the sets $W_1(t)$, $t_0 \leqslant t \leqslant \vartheta$ solves Problem 1.2 with certain $H(N)$ and $G(M)$. However, this is possible only when $x_*(s;\tau) \notin W(t_*)$. The latter property of sets $W^u(t)$, $t_0 \leqslant t \leqslant \vartheta$ follows from the way they were constructed and from Lemma 2.2. The lemma has been proved.

In a manner similar to Lemma 2.5 we can prove

Lemma 2.6. The sets $W^\circ(t)$, $t_0 \leqslant t \leqslant \vartheta$ are strongly u-stable. If $x_0(s;\tau) \in W^\circ(t_0)$, then the strategy U^e extremal to them solves Problem 1.3.

From Lemmas 2.2, 2.5 and 2.6 follow:

Alternative 1. Either $x_0(s;\tau) \in W^u(t_0)$ and then Problem 1.1 is solvable (where as the strategy $U^e(t, x(s;\tau)$ we can take the strategy U^e extremal to the sets $W^u(t)$, $t_0 \leqslant t \leqslant \vartheta$)), or $x_0(s;\tau) \notin W^u(t_0)$, and then Problem 1.2 is solvable.

Alternative 2. Either $x_0(s;\tau) \in W^\circ(t_0)$ and the Probler 3 is solvable (where as the strategy $U^e(t, x(s;\tau)$ we can take the strategy U^e extremal to the sets $W^\circ(t)$, $t_0 \leqslant t \leqslant \vartheta$)), or $x_0(s;\tau) \notin W^\circ(t_0)$ and then Problem 1.4 is solvable.

3. We indicate two methods for constructing strongly $u(v)$-stable sets, similar to [1]. Let us set the inclusion

$$x^{\cdot}(t) \in \Phi(t, x(t+s;\omega)), \quad t_0 \leqslant t \leqslant \vartheta \qquad (3.1)$$
$$\Phi(t, x(s;\omega)) = \bigcap_{v\in Q} F(t, x(s;\omega), v)$$

Lemma 3.1. Let $\Phi(t, x(s;\omega)) \neq \varnothing$ for each pair $\{t, x(s;\omega)\}$ from some domain $D \in [t_0, \vartheta] \times B_\omega$ and let there exist an absolutely continuous solution $x = w(t)$ $(t_0 \leqslant t \leqslant \vartheta)$ of inclusion (3.1), satisfying the conditions

$$w(t_0+s;\omega) = x_0(s;\omega),\ w(\vartheta+s;\mu) \in M(\vartheta)$$
$$\{t, w(t+s;\omega)\} \in D \cap N, \quad t_0 \leqslant t \leqslant \vartheta$$

Also let Condition A be satisfied in some open domain D_1 such that $\{t, w(t+s;\omega)\} \in D_1$, $t_0 \leqslant t \leqslant \vartheta$. Then the strategy U^e extremal to the path

$$W = [\{t, x(s;\tau)\} \mid t_0 \leqslant t \leqslant \vartheta,\ x(s;\tau) = w(t+s;\tau)]$$

ensures the displacement of all positions $\{t, x_\tau[t+s; p_0, U^e]\}$ along this path up to contact with $\{\vartheta, M(\vartheta)\}$.

Let Condition A be satisfied for each pair $\{t, x(s;\omega)\}$ from some open domain D_2, such that $N \subset D_2$ and $\{t_0, x_0(s;\omega)\} \in D_2$ and let the function

$$\varkappa(t, x(s;\omega), l) = -\max_{u\in P}\min_{v\in Q} l'f(t, x(s;\omega), u, v)$$

be convex in l. We denote $x(t) = x(t; t_0, x_0(s;\omega))$ to be an arbitrary solution of the inclusion

$$x^{\cdot}(t) \in R(t, x(t+s;\omega)), \quad t_0 \leqslant t \leqslant \vartheta \qquad (3.2)$$
$$R(t, x(s;\omega)) = \bigcap_{u\in P} F(t, x(s;\omega)), u)$$

and let $\tau = \tau(x(\cdot))$ be the first instant that either $\{\tau, x(\tau+s;\omega)\} \notin D_2$ or $\{\tau, x(\tau+s;u)\} \in M$.

Lemma 3.2. If $R(t, x(s;\omega)) \neq \varnothing$ for $\{t, x(s;\omega)\} \in D_2$ and the solution $x(t) = x(t; t_0, x_0(s;\omega))$ of inclusion (3.2) exists, for which the position

$\{t, x(t+s; \tau)\}$ bypasses M until an exit from N or bypasses M until the instant ϑ, then the strategy V^e extremal to the path $\{t, x(t+s; \tau)\}$ solves Problem 1.2. Otherwise, the strategy U^e extremal to the set $W = [\{t, x(s; \tau)\} \mid t_0 \leqslant t \leqslant \tau(x(\cdot)), x(t) = x(t; t_0, x_0(s; \omega))]$ solves Problem 1.1.

Note 3.1. From these results we can similarly obtain (see [1], Sect. 18) the solutions of the following problems.

Problem 1.5. Find the strategy $U = U_e(t, x(s; \tau))$ satisfying the condition

$$\sup_{x[\cdot]} I(x_{U^e}[\vartheta + s; \mu]) = \min_U \sup_{x[\cdot]} I(x_U[\vartheta + s; \mu])$$

$$(x_U[t] = x[t; \{t_0, x_0(s; \tau)\}, U])$$

Problem 1.6. Find the strategy $V = V_e(t, x(s; \tau))$ satisfying the condition

$$\inf_{x[\cdot]} I(x_{V^e}[\vartheta + s; \mu]) = \max_V \inf_{x[\cdot]} I(x_V[\vartheta + s; \mu])$$

$$(x_V[t] = x[t; \{t_0, x_0(s; \tau)\}, U])$$

The functional $I(x(s; \mu))$ is assumed to be lower (upper) semicontinuous in B_μ.

3.2. Alternatives 1 and 2 are valid also if by the motion $x[t; \{t_0, x(s; \tau)\}, U]$ $(x[t; \{t_0, x(s; \tau)\}, V])$ we mean a continuous function $x[t]$ with the property: sequence $\{\Delta_j\}$ of coverings exists, with $\{\delta_j\} \to 0$ as $j \to \infty$, such that the sequence

$$\{x_{\Delta_j}[t; \{t_0, x_j(s; \tau)\}, U]\}$$

$$(\{x_{\Delta_j}[t; \{t_0, x_j(s; \tau)\}, V]\})$$

of functions converges in $C([t_0, \vartheta])$ to $x[t]$, where $x_j(s; \tau) \to x(s; \tau)$ in B_τ as $j \to \infty$. In this case the sheaf

$$X(\cdot; \{t_0, x(s; \tau)\}, U)\ (X(\cdot; \{t_0, x(s; \tau)\}, V))$$

is upper-semicontinuous by inclusion [1] at each point $x(s; \tau) = x_*(s; \tau)$ (relative to the parameter $x_*(s; \tau)$ and in the metric of $C([t_0, \vartheta]))$. Under such definition of motion the approach-evasion game with information storage [1] (with complete memory [6]) is a special case of Problems 1.1 and 1.2 if the functional $\varphi = \varphi(x[t], t_0 \leqslant t < \infty)$ (see [1], p. 427) satisfies the condition: $\varphi = \varphi(x[t], t_0 \leqslant t \leqslant \vartheta_1)$, where ϑ_1 is some positive number. In this connection

$$\omega = 0,\ \mu = \vartheta_1,\ B_\delta = C([-\delta, 0]),\ N = B_\omega$$

$$(\omega = \mu = \vartheta_1,\ B_\delta = C([-\delta, 0])).$$

The author thanks Iu. S. Osipov for posing the problem and for valuable advice.

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Translated by N. H. C.

UDC 62-50

ON A DIFFERENTIAL-DIFFERENCE GAME OF ESCAPE

PMM Vol. 40, № 6, 1976, pp. 995-1002
A. A. CHIKRII and G. Ts. CHIKRII
(Kiev)
(Received November 10, 1975)

A nonlinear escape problem for conflict-controlled systems described by differential equations with a lagging argument is considered. The sufficient escape conditions which are realized in the class of piecewise-constant functions are obtained. The paper relates to the researches in [1 — 8] and is a continuation of [9, 10].

1. Let a system's motion be described by the differential equation

$$x^{\cdot}(t) = f(x(t), x(t-\tau), u, v), \quad u \in U, \; v \in V \tag{1.1}$$

Here x is the n-dimensional phase vector, u and v are the control parameters of the first and second players, U and V are closed bounded sets. The function $f(x, x_\tau, u, v)$ is continuous in all arguments and is continuously differentiable in x and x_τ, $\tau \geqslant 0$ is the magnitude of the lag. A terminal set M, which is subspace, is delineated in the space E^n. The game terminates if $x(t)$ hits onto set M. As the initial state for the game (1.1) we can take any absolutely continuous function $g(t)$ given in the interval $[-\tau, 0]$. In what follows we assume that derivatives of all the orders needed are present in the functions $g(t)$ used as the intial functions; these derivatives and the functions themselves satisfy in the interval $[-\tau, 0]$ a Lipschitz condition with a constant not exceeding a specified number C.

The vector $x(t)$ moves under the action of the measurable functions $u(t)$ and $v(t)$; the conditions ensuring the continuability of the solution $x(t)$ onto the whole semi-infinite time interval are assumed satisfied. At each instant t the players know the game's state $x_t(s) = x(t+s)$, $-\tau \leqslant s \leqslant 0$. This restricts the information available to the second player from whose position the game is analyzed. We also assume that from the function $x(t)$ specified in some time interval, the escaping can instantly compute its derivatives of all orders needed at any point of the interval.

Let us describe how the game proceeds. From the known current state $x(\cdot)$ (a dot within the parentheses means that the function $x(t)$ on the whole, is being treated as an element of a functional space) the second player determines a number $\varepsilon(x(\cdot)) > 0$, selects a control $v(t) = v(x(\cdot);\; t)$, $0 \leqslant t \leqslant \varepsilon(x(\cdot))$, and informs his opponent. On the basis of the information received the first player sets his own control $u(t)$